Pre-AP Geometry 2-7 Study Guide: Flowchart and Paragraph proofs (pp 118-121)

Attendance Problems. Complete each sentence.

1. If the measures of two angles are _?_, then the angles are congruent.

2. If two angles form a _?_, then they are supplementary.

3. If two angles are complementary to the same angle, then the two angles are _?_.

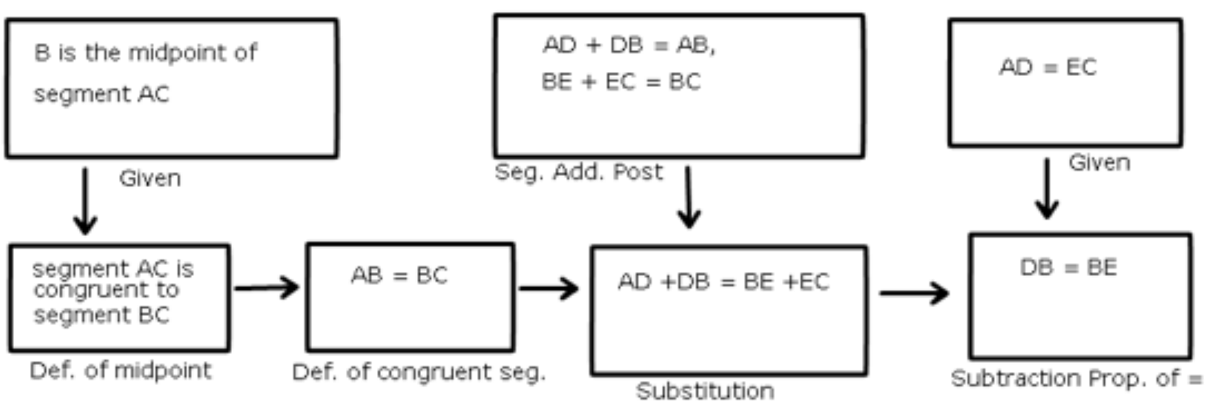
- I can write flowchart and paragraph proofs.
- I can prove geometric theorems by using deductive reasoning.

Vocabulary	
flowchart proof	paragraph proof

Common Core

CC.9-12.G.CO.9 Prove geometric theorems about lines and angles.
CC.9-12.G.CO.10 Prove theorems about triangles.

A second style of proof is a **flowchart proof**, which uses boxes and arrows to show the structure of the proof.



The justification for each step is written below the box.

Q: What do you call two fishermen standing up?
A: Vertical anglers.

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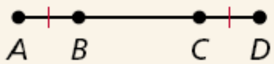
Theorem 2-7-1 Common Segments Theorem

THEOREM	HYPOTHESIS	CONCLUSION
Given collinear points A, B, C, and D arranged as shown, if $\overline{AB} \cong \overline{CD}$, then $\overline{AC} \cong \overline{BD}$.	$\overline{AB} \cong \overline{CD}$	$\overline{AC} \cong \overline{BD}$



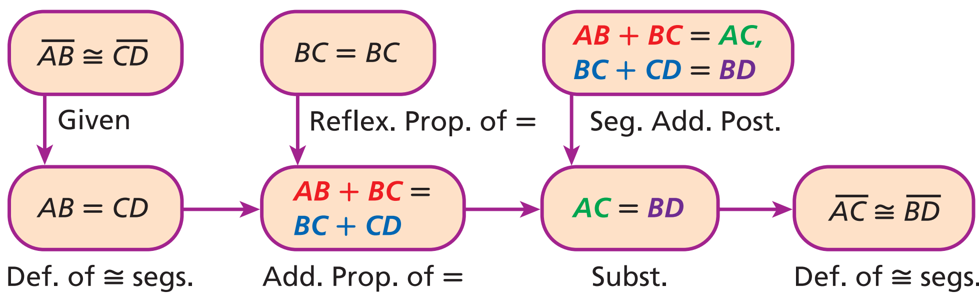
1 Reading a Flowchart Proof

Use the given flowchart proof to write a two-column proof of the Common Segments Theorem.

Given: $\overline{AB} \cong \overline{CD}$

Prove: $\overline{AC} \cong \overline{BD}$

Flowchart proof:



Two-column proof:

Statements	Reasons
1. $\overline{AB} \cong \overline{CD}$	1. Given
2. $AB = CD$	2. Def. of ≅ segs.
3. $BC = BC$	3. Reflex. Prop. of =
4. $AB + BC = BC + CD$	4. Add. Prop. of =
5. $AB + BC = AC$, $BC + CD = BD$	5. Seg. Add. Post.
6. $AC = BD$	6. Subst.
7. $\overline{AC} \cong \overline{BD}$	7. Def. of ≅ segs.

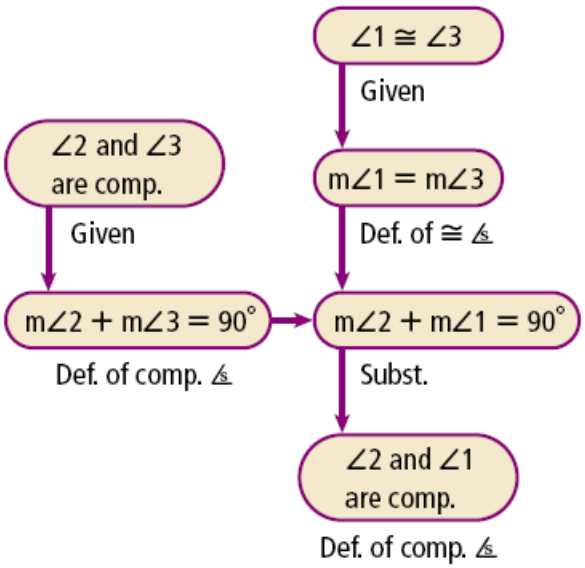
The best that we can do is to be kindly and helpful toward our friends and fellow passengers who are clinging to the same speck of dirt while we are drifting side by side to our common doom. - *Clarence Darrow*

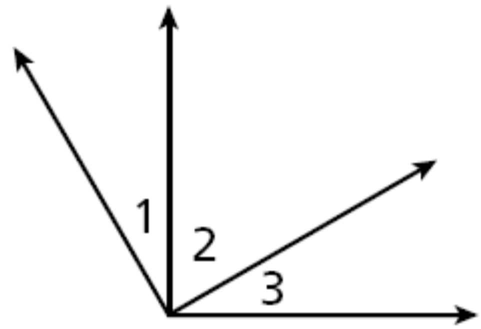
<u>Example 1</u>. Use the given flowchart proof to write a two-column proof.

Given: $\measuredangle 2$ & $\measuredangle 3$ are complementary. $\measuredangle 1 \cong \measuredangle 3$

Prove: $\measuredangle 2$ & $\measuredangle 1$ are complementary.

Flowchart proof:





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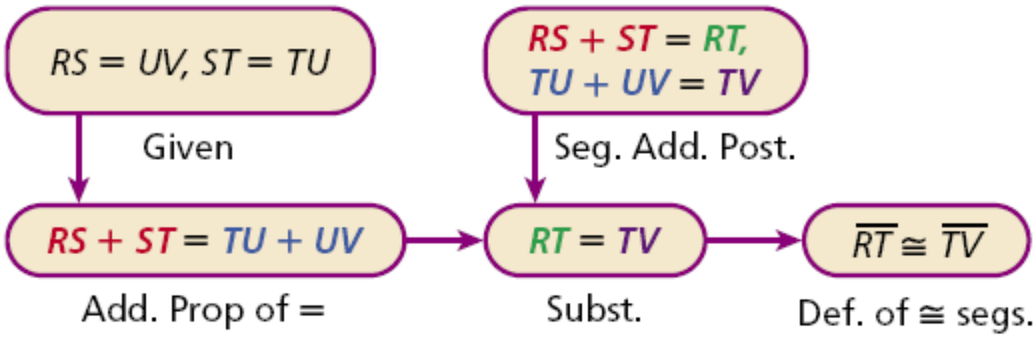
4. Guided Practice: **Use the given flowchart proof to write a two-column proof.**

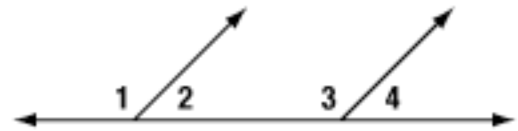
Given: RS = UV, ST = TU

Prove: $\overline{RT} \cong \overline{TV}$

Flowchart proof:



Video Example 2. **Use the given two-column proof to write a flowchart proof.**



Given: $\measuredangle 1 \cong \measuredangle 3$
Prove: $m\measuredangle 2 = m\measuredangle 4$

Statement	Reason
$\measuredangle 1 \cong \measuredangle 3$	Given
$\measuredangle 1$ & $\measuredangle 2$ are supplementary. $\measuredangle 3$ & $\measuredangle 4$ are supplementary.	Linear pair theorem.
$\measuredangle 2 \cong \measuredangle 4$	Congruent supplements theorem.
$m\measuredangle 2 = m\measuredangle 4$	Definition of congruent angles.

2 Writing a Flowchart Proof

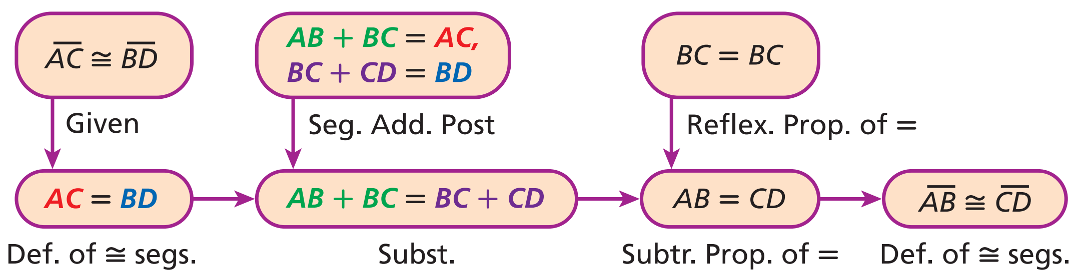
Use the given two-column proof to write a flowchart proof of the Converse of the Common Segments Theorem.

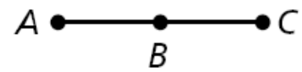
Given: $\overline{AC} \cong \overline{BD}$
Prove: $\overline{AB} \cong \overline{CD}$

Two-column proof:

Statements	Reasons
1. $\overline{AC} \cong \overline{BD}$	**1.** Given
2. $AC = BD$	**2.** Def. of ≅ segs.
3. $AB + BC = AC$, $BC + CD = BD$	**3.** Seg. Add. Post.
4. $AB + BC = BC + CD$	**4.** Subst. *Steps 2, 3*
5. $BC = BC$	**5.** Reflex. Prop. of =
6. $AB = CD$	**6.** Subtr. Prop. of =
7. $\overline{AB} \cong \overline{CD}$	**7.** Def. of ≅ segs.

Flowchart proof:





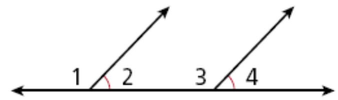
<u>Example 2.</u> Use the given two-column proof to write a flowchart proof.

Given: *B* is the midpoint of $\overline{AC}$.
Prove: $2AB = AC$

Statements	Reasons
1. *B* is the midpoint of $\overline{AC}$.	**1.** Given
2. $\overline{AB} \cong \overline{BC}$	**2.** Def. of mdpt
3. $AB = BC$	**3.** Def. of $\cong$ segs.
4. $AB + BC = AC$	**4.** Seg. Add. Post.
5. $AB + AB = AC$	**5.** Subst.
6. $2AB = AC$	**6.** Simplify

5. Guided Practice. Use the given two-column proof to write a flowchart proof.

Given: $\measuredangle 2 \cong \measuredangle 4$
Prove: $m\measuredangle 1 = m\measuredangle 3$



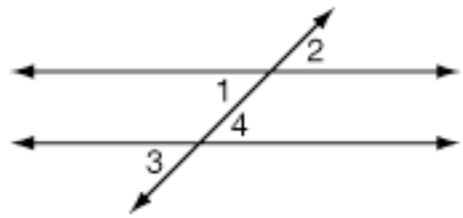
Statements	Reasons
1. $\angle 2 \cong \angle 4$	1. Given
2. $\angle 1$ and $\angle 2$ are supplementary. $\angle 3$ and $\angle 4$ are supplementary.	2. Lin. Pair Thm.
3. $\angle 1 \cong \angle 3$	3. $\cong$ Supps. Thm.
4. $m\angle 1 = m\angle 3$	4. Def. of $\cong$ $\measuredangle$s

A **<u>paragraph proof</u>** is a style of proof that presents the steps of the proof and their matching reasons as sentences in a paragraph. Although this style of proof is less formal than a two-column proof, you still must include every step.

Theorems

THEOREM		HYPOTHESIS	CONCLUSION
2-7-2	**Vertical Angles Theorem** Vertical angles are congruent.	$\angle A$ and $\angle B$ are vertical angles.	$\angle A \cong \angle B$
2-7-3	If two congruent angles are supplementary, then each angle is a right angle. ($\cong$ ∠s supp. $\rightarrow$ rt. ∠s)	$\angle 1 \cong \angle 2$ $\angle 1$ and $\angle 2$ are supplementary.	$\angle 1$ and $\angle 2$ are right angles.

Video Example 3. Use the given paragraph proof to write a two-column proof.



Given: $\measuredangle 1 \cong \measuredangle 3$
Prove: $\measuredangle 2 \cong \measuredangle 4$
Proof: By the Vertical Angle Theorem, $\measuredangle 1 \cong \measuredangle 2$ & $\measuredangle 3 \cong \measuredangle 4$. It is given that $\measuredangle 1 \cong \measuredangle 3$.
By the Transitive Property of Congruence, $\measuredangle 3 \cong \measuredangle 2$, and thus $\measuredangle 2 \cong \measuredangle 4$.

3 Reading a Paragraph Proof

Use the given paragraph proof to write a two-column proof of the Vertical Angles Theorem.

Given: $\angle 1$ and $\angle 3$ are vertical angles.

Prove: $\angle 1 \cong \angle 3$

Paragraph proof: $\angle 1$ and $\angle 3$ are vertical angles, so they are formed by intersecting lines. Therefore $\angle 1$ and $\angle 2$ are a linear pair, and $\angle 2$ and $\angle 3$ are a linear pair. By the Linear Pair Theorem, $\angle 1$ and $\angle 2$ are supplementary, and $\angle 2$ and $\angle 3$ are supplementary. So by the Congruent Supplements Theorem, $\angle 1 \cong \angle 3$.

Two-column proof:

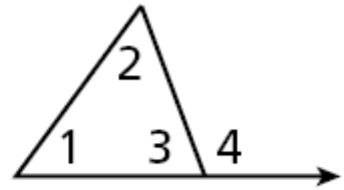
Statements	Reasons
1. $\angle 1$ and $\angle 3$ are vertical angles.	**1.** Given
2. $\angle 1$ and $\angle 3$ are formed by intersecting lines.	**2.** Def. of vert. $\angle$s
3. $\angle 1$ and $\angle 2$ are a linear pair. $\angle 2$ and $\angle 3$ are a linear pair.	**3.** Def. of lin. pair
4. $\angle 1$ and $\angle 2$ are supplementary. $\angle 2$ and $\angle 3$ are supplementary.	**4.** Lin. Pair Thm.
5. $\angle 1 \cong \angle 3$	**5.** $\cong$ Supps. Thm.

PROOF Let $M = (Q, \Sigma, \delta, q_1, F)$ be a DFA recognizing A and p be the number of states of M.

Let $s = s_1 s_2 \cdots s_n$ be a string in A of length n, where $n \geq p$. Let $r_1, \ldots, r_{n+1}$ be the sequence of states that M enters while processing s, so $r_{i+1} = \delta(r_i, s_i)$ for $1 \leq i \leq n$. This sequence has length $n + 1$, which is at least $p + 1$. Among the first $p + 1$ elements in the sequence, two must be the same state, by the pigeonhole principle. We call the first of these r_j and the second r_l. Because r_l occurs among the first $p+1$ places in a sequence starting at r_1, we have $l \leq p+1$. Now let $x = s_1 \cdots s_{j-1}$, $y = s_j \cdots s_{l-1}$, and $z = s_l \cdots s_n$.

As x takes M from r_1 to r_j, y takes M from r_j to r_j, and z takes M from r_j to r_{n+1}, which is an accept state, M must accept xy^iz for $i \geq 0$. We know that $j \neq l$, so $|y| > 0$; and $l \leq p+1$, so $|xy| \leq p$. Thus we have satisfied all conditions of the pumping lemma.

Example 3. Use the given paragraph proof to write a two-column proof.

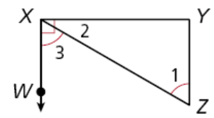


Given: $m\measuredangle 1 + m\measuredangle 2 = m\measuredangle 4$

Prove: $m\measuredangle 3 + m\measuredangle 1 + m\measuredangle 2 = 180°$

Paragraph Proof: It is given that $m\measuredangle 1 + m\measuredangle 2 = m\measuredangle 4$. $\measuredangle 3$ & $\sphericalangle 4$ are supplementary by the Linear Pair Theorem. So $m\measuredangle 3 + m\measuredangle 4 = 180°$ by definition of supplementary. By Substitution, $m\measuredangle 3 + m\measuredangle 1 + m\measuredangle 2 = 180°$

6. Guided Practice. Use the given paragraph proof to write a two-column proof.

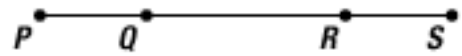


Given: $\measuredangle WXY$ is a right angle. $\measuredangle 1 \cong \measuredangle 3$

Prove: $\measuredangle 1$ & $\measuredangle 2$ are complementary.

Proof: Since $\measuredangle WXY$ is a right angle, $m\measuredangle WXY = 90°$ by the definition of a right angle. By the Angle Addition Postulate, $m\measuredangle WXY = m\measuredangle 2 + m\measuredangle 3$. By substitution, $m\measuredangle 2 + m\measuredangle 3 = 90°$. Since $\measuredangle 1 \cong \measuredangle 3$, $m\measuredangle 1 = m\measuredangle 3$ by the definition of congruent angles. Using substitution, $m\measuredangle 2 + m\measuredangle 1 = 90°$. Thus by the definition of complementary angles, $\measuredangle 1$ & $\measuredangle 2$ are complementary.

Video Example 4. Use the given two-column proof to write a paragraph proof.



Given: $\overline{PQ} \cong \overline{RS}$

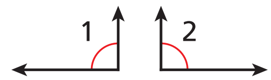
Prove: $\overline{PR} \cong \overline{QS}$

Statement	Reason
$\overline{PQ} \cong \overline{RS}$	Given
PQ = RS	Definition of congruent segments.
QR = QR	Reflexive property of equality.
PQ + QR = RS + QR	Addition property of equality.
PQ + QR = PR RS + QR = QS	Segment addition postulate.
PR = QS	Substitution property of equality.
$\overline{PR} \cong \overline{QS}$	Definition of congruent segments.

4 Writing a Paragraph Proof

Use the given two-column proof to write a paragraph proof of Theorem 2-7-3.

Given: $\angle 1$ and $\angle 2$ are supplementary. $\angle 1 \cong \angle 2$
Prove: $\angle 1$ and $\angle 2$ are right angles.

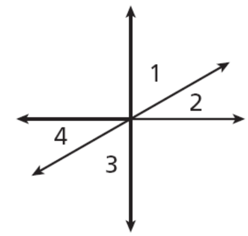


Two-column proof:

Statements	Reasons
1. $\angle 1$ and $\angle 2$ are supplementary. $\angle 1 \cong \angle 2$	**1.** Given
2. $m\angle 1 + m\angle 2 = 180°$	**2.** Def. of supp. $\angle$s
3. $m\angle 1 = m\angle 2$	**3.** Def. of $\cong$ $\angle$s *Step 1*
4. $m\angle 1 + m\angle 1 = 180°$	**4.** Subst. *Steps 2, 3*
5. $2m\angle 1 = 180°$	**5.** Simplification
6. $m\angle 1 = 90°$	**6.** Div. Prop. of =
7. $m\angle 2 = 90°$	**7.** Trans. Prop. of = *Steps 3, 6*
8. $\angle 1$ and $\angle 2$ are right angles.	**8.** Def. of rt. $\angle$

Paragraph proof: $\angle 1$ and $\angle 2$ are supplementary, so $m\angle 1 + m\angle 2 = 180°$ by the definition of supplementary angles. They are also congruent, so their measures are equal by the definition of congruent angles. By substitution, $m\angle 1 + m\angle 1 = 180°$, so $m\angle 1 = 90°$ by the Division Property of Equality. Because $m\angle 1 = m\angle 2$, $m\angle 2 = 90°$ by the Transitive Property of Equality. So both are right angles by the definition of a right angle.

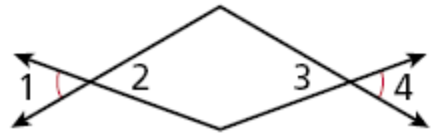
Example 4. Use the given two-column proof to write a paragraph proof.



Given: $\measuredangle 1 \,\&\, \measuredangle 2$ are complementary.
Prove: $\measuredangle 3 \,\&\, \measuredangle 4$ are complementary.

Statement	Reason
$\measuredangle 1 \,\&\, \measuredangle 2$ are complementary.	Given
$m\measuredangle 1 + m\measuredangle 2 = 90°$	Definition of complementary angles.
$\measuredangle 1 \cong \measuredangle 3$ $\measuredangle 2 \cong \measuredangle 4$	Vertical angles theorem.
$m\measuredangle 1 = m\measuredangle 3$ $m\measuredangle 2 = m\measuredangle 4$	Definition of congruent angles.
$m\measuredangle 3 + m\measuredangle 4 = 90°$	Substitution property of equality.
$\measuredangle 3 \,\&\, \measuredangle 4$ are complementary.	Definition of complementary angles.

7. Guided Practice. Use the given two-column proof to write a paragraph proof.



Given: $\angle 1 \cong \angle 4$
Prove: $\angle 2 \cong \angle 4$
Two-column proof:

Statements	Reasons
1. $\angle 1 \cong \angle 4$	**1.** Given
2. $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$	**2.** Vert. ∠s Thm.
3. $\angle 2 \cong \angle 4$	**3.** Trans. Prop. of ≅ *Steps 1, 2*
4. $\angle 2 \cong \angle 3$	**4.** Trans. Prop. of ≅ *Steps 2, 3*

<u>2-7 Flowchart and Paragraph proofs</u>

- *(p 123)* 7-10, 12, 14, 16-20, 24, 26.
- 2B Ready to Go On & posttests.